

Quantitative Aptitude Problems on Ages and Averages

Averages:

What is average?

The concept of average is equal distribution of the overall value among all the things or persons present there. So the formula for finding the average is as follows:

Average, $A = \text{Total of all things, } T / \text{Number of things, } N$

Therefore, Total, $T = AN$

If any person joins a group with more value than the average of the group then the overall average increases. This is because the value in excess than the average will also be distributed equally among all the members.

Similarly when any value less than the average joins the group the overall group decreases as the deficit is divided equally among all the people present there.

Example:

Consider three people A, B and C with total of Rs. 30/-. Their average becomes Rs. 10/- for each. If another person D joins them with Rs. 50/- then he has Rs. 40/- more than actual average of Rs. 10/-.

So this Rs. 40/- will get distributed among those four and each gets Rs. 10/-. Thus the average becomes Rs. 20/- each.

Example:

The average age of a class of 30 students is 12. If the teacher is also included the average becomes 13 years. Find the teacher's age.

Soln:

When the teacher is included there are totally 31 members in the class and the average is increased by 1 year. This means that everyone got 1 extra year after distributing the extra years of the teacher.

So extra years of the teacher are as follow: $31 \times 1 = 31$ years.

Age of the teacher = actual avg + extra years = $12 + 31 = 43$ years

Quantitative aptitude Problems on Averages and ages:

1. The average of 13 papers is 40. The average of the first 7 papers is 42 and of the last seven papers is 35. Find the marks obtained in the 7th paper?

- (A) 23
- (B) 38
- (C) 19
- (D) None of these

2. The average age of the Indian cricket team playing the Nagpur test is 30. The average age of 5 of the players is 27 and that of another set of 5 players, totally different from the first five, is 29. If it is the captain who was not included in either of these two groups, then find the age of the captain.

- (A) 75
- (B) 55
- (C) 50
- (D) Cannot be determined

3. A bus goes to Ranchi from Patna at the rate of 60 km per hour. Another bus leaves Ranchi for Patna at the same time as the first bus at the rate of 70 km per hour. Find the average speed for the journeys of the two buses combined if it is known that the distance from Ranchi to Patna is 420 kilometers.

- (A) 64.615 kmph
- (B) 64.5 kmph
- (C) 63.823 kmph
- (D) 64.82 kmph

4. A train travels 8 km in the first quarter of an hour, 6 km in the second quarter and 40 km in the third quarter. Find the average speed of the train per hour over the entire journey.

- (A) 72 km/h
- (B) 18 km/h
- (C) 77.33 km/h
- (D) 78.5 km/h

5. The average weight of 6 men is 68.5 kg. If it is known that Ram and Sham weigh 60 kg each, find the average weight of the others.

- (A) 72.75 kg
- (B) 75 kg
- (C) 78 kg
- (D) None of these

6. The average score of a class of 40 students is 52. What will be the average score of the rest of the students if the average score of 10 of the students is 61.

- (A) 50
- (B) 47
- (C) 48
- (D) 49

7. The average age of 80 students of IIM, Bangalore of the 1995 batch is 22 years. What will be the new average if we include the 20 faculty members whose average age is 37 years?

- (A) 32 years
- (B) 24 years
- (C) 25 years

(D) None of these

8. Out of the three numbers, the first is twice the second and three times the third. The average of the three numbers is 88. The smallest number is

(A) 72

(B) 36

(C) 42

(D) 48

9. The sum of three numbers is 98. If the ratio between the first and second is 2 : 3 and that between the second and the third is 5 : 8, then the second number is

(A) 30

(B) 20

(C) 58

(D) 48

10. The average height of 30 girls out of a class of 40 is 160 cm and that of the remaining girls is 156 cm. The average height of the whole class is

(A) 158 cm

(B) 158.5 cm

(C) 159 cm

(D) 157 cm

The problems, which 10 are from different models, try to solve them. while solving the problems on averages and ages in your exam, answer them in 40-45 sec, because of they are basically very easy problems and also the concepts of averages and ages are same.

An Average is a measure that tells us information about a group of things rather than data about individuals.

Mean, median, and mode are all types of averages, although the mean is the most common type of average and usually refers to the arithmetic mean. There are other kinds of means that are more difficult.

The arithmetic mean is a simple type of average. Suppose you want to know what the numerical average is in your math class. Let's say your grades so far are 80, 90, 92, and 78 on the four quizzes you have had. To find your quiz average, add up the four grades: $80 + 90 + 92 + 78 = 340$

Then divide that answer by the number of grades that you started with, four: $340/4 = 85$. So, your quiz average is 85. Whenever you want to find a mean, just add up all the numbers and divide by the number of numbers you started with.

But sometimes the arithmetic mean doesn't give you all the information you want. Suppose you are looking for a job. You interview with a company that has ten employees, and the interviewer tells you that the average salary is Rs. 20000 per month. That's a lot of money! But that's not what you would be making.

For this particular company, you would make half of that. Each employee makes Rs.10000 per month, except for the owner, who makes Rs.110000 per month. Nine employees make Rs.10000, so adding those up is $9 \times 10000 = 90000$. Then the owner makes Rs.110000, so the total is $Rs.110000 + Rs.90000 = Rs.200000$. Divide by the total number of employees, ten, and we have $Rs.200000/10 = Rs.20000$. Because the owner makes so much more than everyone else, her salary "pulls" the average up.

A better question to ask is, "What is the median salary?" The median is the number in the middle, when the numbers are listed in order. For example, suppose you wanted to find the median of the numbers 6, 4, 67, 23, 6, 98, 8, 16, 37. First, list them in order: 4, 6, 6, 8, 16, 23, 37, 67, 98. Now, which one is in the middle? Well, there are nine numbers, so the middle one is the fifth, which is 16, so 16 is the median.

Now, what about when there is an even number of numbers? Look at the quiz grade example again: 90, 80, 92, 78. First list the numbers in order: 78, 80, 90, 92.

The two middle ones are 80 and 90. So do we have two medians? No, we find the mean of those two: $80 + 90 = 170$, and $170/2 = 85$. So 85 is the median.

Now look at those salaries again. To find the median salary, we look at the salaries in order: 10000, 10000, 10000, 10000, 10000, 10000, 10000, 10000, 10000, 110000. This is an even number of salaries, so we look at the middle two. They are both 10000, so the median is Rs.10000. That's much better at telling you how much you'll make if you accept the job.

But the median doesn't always give you the best information either. Suppose you interview with a company that has 10 general employees, 7 assistants, 3 managers, and 1 owner. For this company, the mean salary could be Rs.5000, and the median Rs. 4000. But you are applying for the position of general employee, whose starting salary is Rs. 1000.

The mode is the type of average you want to know in this situation. The mode is the number that occurs most frequently. In the example for median, 6 would be the mode because it occurs twice, while the other numbers each occur once. In our employee example, the mode is Rs. 10000 because that number occurs ten times, which is more than the number of times any other number occurs.

The most common value found in a group consisting of several readings is called as the 'mode'. A group of observation can have more than one modal value.

± Mean, median and mode are all good types of averages, and each works best in different types of situations.

In the following example, we will measure the central tendency (that is mean; median, and mode values) of the given group of readings; and later we also will try to understand how these measures of central tendency may change when we add more values to the group of readings.

Example:

A person wrote down the time he spent on the exercises in a week. These readings are as follows.

Day	Exercise time
Monday	20 min.
Tuesday	30 min.

Wednesday	20 min.
Thursday	35 min.
Friday	25 min.

The graph of the data is as shown below.

GRAPH

1. Now we calculate the mean.

Mean = Sum of the readings/Number of readings = $20 + 30 + 20 + 35 + 25/5$
Therefore Mean = $130/5 = 26$ minutes

Now we calculate the median, after arranging the given set of numbers in the increasing order we get,

Exercise time: 20, 20, 25, 30, and 35

The median (middle) value is 25 minutes.

Now we calculate the modal value. To calculate the mode we need to see how many times each number is repeating. And then the mode is the number that is repeating for the maximum.

Number 20 is repeating 2 times. Numbers 25, 30 and 35 are repeating only once. Therefore the mode is 20 minutes.

- \pm the mean, median and mode must have the same unit that is associated with the given set of readings.

Let's say the person adds Saturday in the exercise schedule. And on Saturday 30 minutes are spent on the exercise. To calculate the new mean value, add up all the readings and divide the sum by total number of readings.

Mean = $20 + 30 + 20 + 35 + 20 + 30/6 = 160/6 = 26.66$ minutes

Or, what we can do is multiply the original mean by the original number of readings to get the sum of the original readings.

$$26 \times 5 = 130$$

We add the new value of 30 to this to get 160 and divide it by 6 to get the new mean. To calculate the new median, we rearrange the numbers in the increasing order. Therefore we now get,

Exercise time: 20, 20, 25, 30, 30, and 35

However in this case we do not have one single middle value. Instead we have two numbers in the middle that are 25 and 30. In this case we take the average of the two middle numbers to find the median. Therefore average of 25 and 30 is $(25+30)/2 = 27.5$ minutes.

Therefore the new median value is 27.5 minutes.

To calculate the new mode, we observe that numbers 20 and 30 minutes repeat 2 times. Numbers 25 and 35 minutes repeat only once. Therefore this set of readings has two mode values and the two modes are 20 and 30 minutes.

Example: Find the measures of central tendency for the following readings.

-2, 350%, 12, (-3)², -80%

Solution: To compute the mean, median, or mode all the readings must be presented in the same number form.

Therefore we will need to convert the percentages into decimal form.

Therefore after converting the given set of readings into same number form, the new set has the following values.

-2, 3.5, 12, 9, -0.8

Now, mean = $-2 + 3.5 + 12 + 9 - 0.8/5 = 21.7/5 = 4.34$

Now median of the readings is the middle value of the set after arranging the numbers in the increasing order, which is

-2, -0.8, 3.5, 9, 12

Since none of the numbers repeats more than once, the above set of readings has no mode.

The concept of averages can be applied to problems in slightly different way.

Example: During the week, John watches TV every night after he finishes his homework. The following table shows how much time he spent last week watching television.

Day	Entertainment time
Monday	1 hour
Tuesday	30 min.
Wednesday	30 min.

Thursday	45 min.
Friday	2 hours

John decided to include his TV time for Saturday and Sunday as well. When he did and recalculated his mean, mode and median he discovered that only the mean had changed. What can we say about the TV time for Saturday and Sunday that would produce this situation?

Solution: To proceed with the numbers, they must be converted to the same units. After rearranging the numbers to get the increasing order for entertainment time, we have: 30 min, 30 min, 45min, 60min, 120 min

Therefore, mean = $285/5 = 57$ min and median = 45 min.

Since number 30 repeats 2 times and other numbers repeat only once, we can say that mode = 30 min.

We need to find additional possible values for Saturday and Sunday that will not change the median and mode.

Let us look at the number sequence.

30, 30, 45, 60, 120

If 45 should remain as the median (in the middle), then if we write a number to its left, we must write a number to its right side also. And if number 30 should remain as the mode, then the two more numbers, representing Saturday and Sunday, that we will include into to the data set must be different from 45, 60 and 120.

Therefore we can say if 45 minutes should remain the median value for the entire week, then if John spends more than 45 minutes but neither 1 hour nor 2 hours watching TV on Saturday, then on Sunday he must spend less than 45 minutes watching television. (The amount of time spent on TV on Saturday and Sunday can be swapped).

Average properties

1. If each number in a set of data points is increased by a certain number x , then the average will be increase by x , similarly for incase of decrease it will decrease by x , and in case of multiplication it will be multiplied by x .

Example: In a class of ten students, where the average weight is 50 kgs, if weight of each student is increased by 5 kg what will be the new average?

Number of students = 10, average = 50,

Total weight of class = $500 (50 \times 10)$

In case each student's weight is increased by 5 kg, excess weight = $50 (5 \times 10)$

Total weight of the class will become = $500 + 50 = 550$

Average weight = 55 , so average is also increased by 5 kgs

2. The average speed in case of a journey from X to Y at speed of A m/sec and returning back to X at a speed of B m/sec, is m/sec

Example: Sunil travels from Delhi to Noida at the speed of 40 km/hr and returns at the speed of 50 km/hr. What is the average speed of the journey?

Using the formula,

=

= 44.44 Km/hr

Always remember the units, in what units you have started and in what units you are ending.

Alternative way is by the basic principle of mechanics,

Speed =

Which means Time =

And Average Speed =

Suppose distance from Delhi to Noida is A, therefore

Time while going, $T_1 =$

Time while coming $T_2 =$

Total time =

Total Distance = $A + A = 2A$

Average speed =

=

=

= 44.44 Km/hr

Here are some concepts of various kinds of mean, students are requested to go through the concepts and understand the applications rather than cramming the formulas. There is no direct question expected in CAT on basis of these formulas, But application may be used widely in many concepts especially data interpretation.

Arithmetic mean: As the definition given above, The AM of n numbers $X_1, X_2, X_3, \dots, X_n$ is denoted by \bar{X} and calculated as

$\bar{X} =$

$\bar{X} =$

The Σ is called sigma, which represents summation, from values 1 to n.

For many values it is difficult to calculate mean using this formula. There is an easier formula for that using two concepts, Assumed mean (a) and Deviation (d) from the Assumed mean

$\bar{X} =$

Here $d_i = X_i - a$

Example: AM of 20, 30, 40, 52, 96, 34, 36. Let the assumed mean be 40

X_i d_i

20 $20 - 40 = -20$

30 $30 - 40 = -10$

40 $40 - 40 = 0$

52 $52 - 40 = 12$

96 $96 - 40 = 56$

34 $34 - 40 = -6$

36 $36 - 40 = -4$

$\Sigma d_i = 28$

Mean, $\bar{X} = 40 + \frac{28}{7} = 40 + 4 = 44$

Weighted AM: If there are two values, a and b, have weights (or frequency) of x and y, then the weighted arithmetic mean is given by

WAM =

What is “weight” in weighted mean?

The weight is the relative strength of a data point, lets take respective an example:

Strength: Suppose there is a paper with four sections of 100 marks each – Maths, English, Social studies and History. Now there are different weightages assigned to each that is Maths – 40%, English – 20%, Social studies – 20% and History – 20%. If a student gets following marks (Maths –60, English – 60, Social studies – 50 and History – 50) in different subjects, what are the net marks out of 100?

Here to get net marks out of 100, the respective marks in subjects have to be treated with its weights as per following table:

Subject Marks Net marks
(marks*weight)

Maths 60 0.4(40%) 24

English 60 0.2(20%) 12

Social studies 50 0.2(20%) 10

History 50 0.2(20%) 10

Total (out of 100) – Mean Marks 56

Let's also understand what is frequency?

Frequency: The concept of frequency is not the same as weight but can be treated in the same manner. Suppose the marks in English for a class of 10 people are 50, 60, 50, 51, 65, 60, 50, 36, 60 and 65. Here 50 have occurred 3 times, 60 has occurred three times etc, so the times a data point occurs in a set of data points is called its frequency. Now for the above give data points, the following is the frequency and mean calculation:

Marks Frequency Total

50 3 150

60 3 180

51 1 51

65 2 130

36 1 36

Total 10 547

So the mean here is = 54.7

If Weights W_1, W_2, \dots, W_n are assigned to the values X_1, X_2, \dots, X_n , then the Weighted AM

$WAM =$

Example: WAM of 10, 15, 20, 25 with frequency 1, 2, 3, 5

$X_i \quad W_i \quad W_i X_i$

10 1 10

15 2 30

20 3 60

25 5 125

10 225

$\sum W_i = 10 \quad \sum X_i W_i = 225$

$WAM = 22.5$

Mean of means: If the means (M_1, M_2, \dots) are given along with numbers (N_1, N_2, \dots), then the mean of means is given by using the Weighted Arithmetic Mean formula, where weights are replaced by respective number:

Mean of means =

Example: The mean marks scored by 60 boys are 80 and those scored by 100 girls are 60

Mean = = = 67.5 Marks

Geometric mean: The other mean is the geometric mean, which is given by n th root of the product of n numbers.

GM of $A_1, A_2, A_3, \dots, A_n$ =

Example: GM of 2 and 8 is = = 4

Properties

1. The AM of given set of data points is always equal to or greater than their GM, that is $AM \geq GM$

Example: Find the Geometric mean and Arithmetic mean of 2, 8

Arithmetic Mean = = 5

Geometric Mean = = = 4

2. If the geometric mean of one group of “a” numbers is x and that of another group of “b” numbers is y , then the geometric mean of the combined groups is:

Harmonic mean: The Harmonic mean is give by

For two numbers a and b Harmonic mean is given by

HM =

= =

For N numbers ($a, b, c \dots n$),

HM =

Harmonic mean is used to calculate average speed, with different speeds for same distance.

Example: Ajay travels from Delhi to Shimla at the speed of 50 km/hr and returns at the speed of 40 km/hr, what is the average speed of the journey?

Using the formula of Harmonic Mean,

=

= = 44.44 Km/hr

Weighted Harmonic Mean: The Weighted Harmonic mean is give by

For two numbers a and b with weights x and y it is given by

Weighted HM =

This is used to calculate the average speed when different distances are covered at different speeds.

Example: A man covers his first 6 km at an average speed of 10 m/s, another 4 km at 8 m/s and the last 3 km at 5 m/s.

Average speed = weighted HM =

Mode: It is the number that occurs most frequently in a given set of numbers.

Example: In set of numbers, 1, 2, 3, 4, 5, 5, 4, 3, 2, 1 and 5, the number 5 occurred the maximum number of times, so the mode is 5.

Median: It is the middle value of a set of data points arranged in an ascending or descending order.

If the number of data points is odd, the median is value.

If the number of data points is even, the median is give by the average of the two middle values, suppose the medium values are a and b, here median is given by .

Here

$a = \text{and } b = + 1$

Example: Median of numbers 25, 22, 30, 29, 36

First the values need to be arranged in the ascending order 22, 25, 29, 30, 36

Median is the value P 3rd value, which is 30

Example: Median of numbers 25, 22, 30, 29, 36, 38

First the values need to be arranged in the ascending order 22, 25, 29, 30, 36, 38

The two middle values are $= 3$ and $+ 1 = 4$, the 3rd and 4th values are 29 and 30

Thus median $= = = 28$.